Astro 201; Project Set #6
due 4/27/2012

A problem on synchrotron emission

Powering Radio Lobes

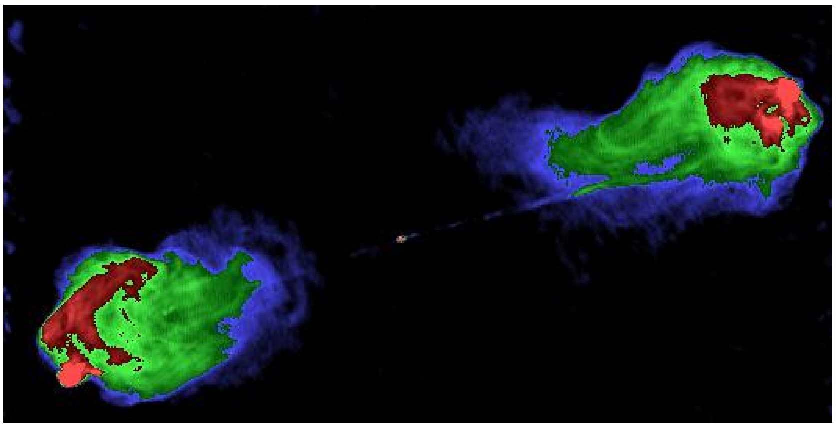

Figure 1: Radio image of Cygnus A, showing the extended lobes powered by narrow jets from the central super-massive black hole. The lobes are about 30 kpc in radius, and the emission is strongest at the front regions which are presumably interacting with the surrounding medium

THE GALAXY CYGNUS A is one of the most powerful radio sources in the sky. Striking radio images (figure 1) reveal a pair of immense lobes of emission, which sit about $\sim$ 100 kpc outside the central galaxy. It is thought that these lobes are powered by a super-massive black hole (SMBH) at the galactic center. The SMBH somehow produces narrow, relativistic jets, which propagate to the outskirts of the galaxy, interact with the ambient medium, and form shocks. The radio luminosity is the result of synchrotron emission from relativistic electrons produced in those shocks.

Figure 2 shows the observed radio spectrum of Cyg A, which resembles what we might expect from synchrotron emission – broad-band, roughly power-law, clearly non-thermal. The observed flux increases down to a frequency of at least 10 MHz, where the value is $F_\nu \approx 10^4$ Jy[1]. In this region, one can reasonably fit a power law to the spectrum and write the specific luminosity

$$L_\nu = 4\pi d^2 F_\nu = 5 \times 10^{36} \left(\frac{\nu}{10\text{ MHz}}\right)^{-0.8} \text{ergs s}^{-1}\text{ Hz}^{-1} \qquad (1)$$

where we have taken the distance to Cyg A to be $d \approx 230$ Mpc. Integrating equation 1 over the range $10^7 - 10^{11}$ Hz gives a luminosity of $L \sim 10^{45}$ ergs s^{-1} (about 1000 times the energy radiated by a supernova at peak).

[1] The Jansky, defined by 1 Jy $= 10^{-23}$ ergs s^{-1} Hz^{-1} cm^{-2} is the preferred measurement of flux in radio astronomy.

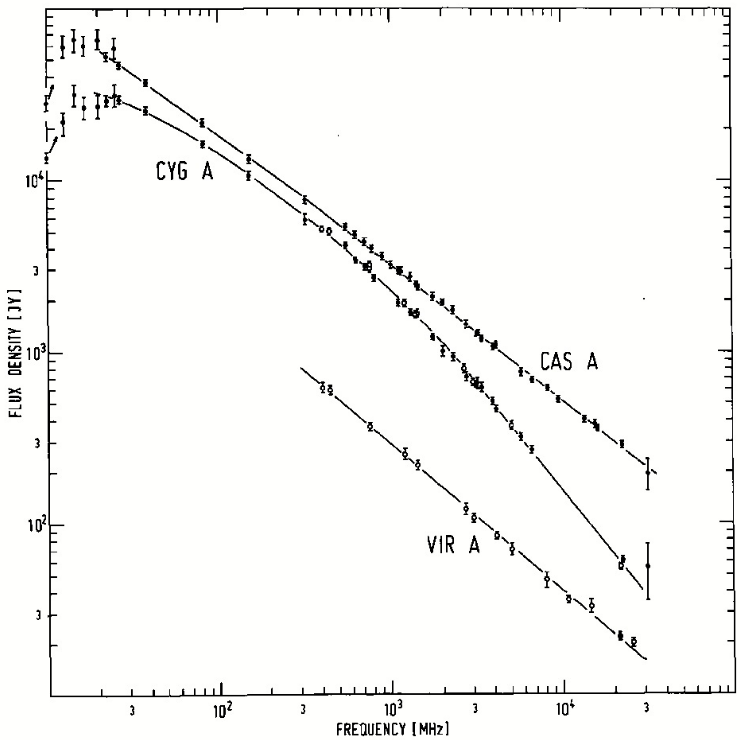

Figure 2: Radio spectrum of Cyg A (and other objects) from Baars et al (1977)

What are the energetics and magnetic field strengths involved in these luminous radio lobes, and can a SMBH really provide the necessary oomph? The total energy[2] in the lobes can be written

$$E = (u_e + u_B) \times 2V \tag{2}$$

where u_e, u_B, are the electron and magnetic field energy densities, respectively, and V is the volume of a lobe[3]. We do not have enough information to directly determine these energy densities, but we can follow the famous arguments of Geoffery Burbidge to derive a lower limit. The implied energies, we will find, are massive – the equivalent of a billion supernova explosions or more.

As usual, we'll take the electron number density to be described by a power law distribution in lorentz factor

$$n d\gamma = C\gamma^{-p} d\gamma \tag{3}$$

where C is some constant and the power-law cuts off at some lower value, $\gamma_{\rm min}$.

a) Infer the value of p from the observations.

b) Integrate equation 3 to get an expression for the energy density of electrons, u_e in terms of C and $\gamma_{\rm min}$.

[2] We are going to ignore the energy in protons, since there is no easy way to measure them directly. But presumably there is as much or more energy in protons than electrons.

[3] The lobes each have a radius of around 30 kpc. We have included a factor of 2 in the equation since there are two of them

What should we use for $\gamma_{\rm min}$ and C? It looks like the observed spectrum might be peaking at $\nu_m \approx 10$ MHz. We can therefore associate[4] 10 MHz with the critical frequency of electrons with lorentz factor $\gamma_{\rm min}$. To determine the constant C, we can use the observed specific luminosity at the frequency $\nu_m = 10$ MHz. From synchrotron theory we know that the specific luminosity (ergs s^{-1} Hz^{-1}) from a power law distribution of electrons is

$$L_\nu \approx \frac{2C}{3} \frac{u_B \sigma_t c}{\nu_{\rm cyc}} \left(\frac{\nu}{\nu_{\rm cyc}} \right)^{-(p-1)/2} \times V \tag{4}$$

[4] This may or may not be correct. The data is getting a little sketchy around 10 MHz – perhaps with better quality observations we would find that the power law extends below 10 MHz. It could also be that the turn over is real, but due to synchrotron self-absorption, not to the minimum lorentz factor. If ν_m is in fact smaller than 10 MHz, this would imply an even smaller $\gamma_{\rm min}$ and hence even more energy. Thus what follows is at least a lower limit to the energy.

c) Show that the energy density of electrons can be written

$$u_e = A \frac{L_m \nu_m^{1/2}}{V} B^{-3/2} \tag{5}$$

where L_m is the observed specific luminosity at ν_m, and A is some combination of numerical factors and fundamental constants.

To proceed, we need to know the magnetic field strength. Unfortunately, there is no easy way to measure this directly. What is typically done instead is to make a *minimum energy* argument. The magnetic energy density increases with B, whereas equation 5 shows that u_e grows as B *decreases.* Thus there must be some comprise value of B which minimizes the total energy. This value will at least give us a lower limit to the necessary energetics.

d) Show that the total energy is minimized when $u_B = 3/4 u_e$ – that is, when u_e and u_B are roughly equal. For this reason, the minimum energy argument is often also called an equipartition argument.

e) Adopting the equipartition argument above, what is (numerically) the magnetic field strength in the emitting regions? What is the minimum lorentz factor, $\gamma_{\rm min}$?

f) What is the total energy in the radio lobes? How does this compare to the typical energy of a supernova?

g) How much mass mass would a black hole have to eat in order to create this energy? Assume that the rest mass energy of the accreted material is processed into jet energy with 10% efficiency. Does the SMBH hypothesis hold together? – i.e., is your estimate for the swallowed mass consistent with SMBH masses?

Comment: We have provided no physical justification for why this system should generate magnetic fields with $u_B \approx u_e$. We have only shown that equipartiation is the optimal configuration for radiating efficiently. However, if the magnetic field is produced by turbulent motions in the plasma, which also play a role in accelerating the high energy particles, perhaps there is some rationale for thinking that we may reach something close to equipartition. This conjecture can now be tested by detailed particle-in-cell simulations, which follow the motions of individual particles in a plasma while simultaneously solving Maxwell's equations to determine the fields they generate. In the absence of any better information, people often simply assume an equipartition B-field in order to carry out synchrotron analyses. Note that we have also neglected here the energy in relativistic protons, which could exceed that in electrons and magnetic fields.